

# Gribov Propagator and Symmetry Breaking: a toy model

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## Abstract

The aim of this paper is to present a connection between the Gribov-Zwanziger condition for the mass gap and spontaneous symmetry breaking. In order to clarify these relationship a toy model is presented and some quantum aspects are discussed.

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# 1 Introduction

One of the most challenging issues in quantum field theory is the understanding of the nonperturbative effects which govern the infrared behavior of quantum field theories, this question is particularly important in the case of Yang-Mills theory. Many mechanisms have been proposed in an attempt to understand, at least partially, this infrared limit. We emphasize two mechanisms in particular, the lattice field theories [1, 2] and the Gribov-Zwanziger mechanism. Recent lattice results have provided evidences of the fact that the infrared regime of the theory is very different from the ultraviolet one and indicates not only a mass gap but a Gribov type propagator [3, 4, 5].

The Gribov-Zwanziger framework [6, 7, 8, 9, 10, 11] consists in restricting the domain of integration in the Feynman path integral within the Gribov horizon. This restriction demands the introduction of a mass parameter and a mass gap equation and this is a key ingredient in the Gribov mechanism [9, 10, 12]. Thus the restriction to the Gribov region is implemented only if in addition to an action that can implement this restriction we can also provide a gap equation to the Gribov parameter.

The achievement of the gap equation is far from easy in the case of a general gauge choice and even more difficult to achieve if we use the idea of location contained in the formulation of Zwanziger in a broader sense, for example in the location of nonlocal operators in fermionic actions. In this sense we want through a toy model to obtain and study the Gribov-Zwanziger mechanism, in particular the setting of the Gribov parameter without the need for conditions other than that the effective potential has a stable minimum [13]. Clearly the stability of the effective potential depends on the effective coupling constant is less than one. This important point will be discussed concurrently with the minimum of the potential.

The aim of this paper is to construct a toy model that presents a gribov type propagator and an effective potential for a mass dimension 2 condensate that can fix the Gribov mass gap due to a non-vanishing expectation value of these condensate. It is important to emphasize that when dealing with a toy model we can not expect all the properties present in the Gribov-Zwanziger action. For example the beta function of Yang-Mills fields is different from that present in this toy model. It is quite clear that an action for scalar fields has no asymptotic freedom, a well known fact in the literature. However as we are dealing with a toy model, in fact as it is also made in [14], we can focus on common points between the model and the Gribov-Zwanziger action.

The paper is organized as follows. In chapter 2 the toy model is presented and the Local composite operator (*LCO*) is introduced, also the BRST symmetries are presented. In Chapter 3 and 4 the equations compatible with the quantum action principle (*QAP*) are presented and the algebraic renormalizability of the model with the insertion is done. In Chapter 5 the effective potential is discussed and parameters are fixed by the renormalization group equation. Chapter 6 is dedicated to present the relation between symmetry breaking and nonlocal BRST symmetries. Finally in the last chapter conclusions are presented.

## 2 The toy model, the LCO formalism and comparison with the Gribov-Zwanziger action

Let us begin by giving the expression for a scalar  $O(n)$  model that presents the insertion of the operator  $\varphi^a \frac{1}{\partial^2} \varphi^a$  in a localized form using auxiliar fields  $\chi, \bar{\chi}, \omega, \bar{\omega}$ , and sources  $J, Q$

$$\begin{aligned} \Sigma = & \int d^4 x_E \left\{ \frac{1}{2} \partial_\mu \varphi^a \partial^\mu \varphi^a + \frac{1}{2} m^2 \varphi^a \varphi^a + \frac{\lambda}{4!} (\varphi^a \varphi^a)^2 + \partial_\mu \bar{\chi}^a \partial^\mu \chi^a - \partial_\mu \bar{\omega}^a \partial^\mu \omega^a \right. \\ & \left. + J^{ab} (\bar{\chi}^a - \chi^a) \varphi^b + Q^{ab} \omega^a \varphi^b - Q^{ab} (\bar{\chi}^a - \chi^a) f_{bcd} \varphi^c \theta^d + \frac{\xi}{2} J^{ab} J^{ab} \right\}, \end{aligned} \quad (1)$$

the action (1) is left invariant under the following set of BRST transformations:

$$\begin{aligned} s\varphi^a &= f_{abc} \varphi^b \theta^c, & s\theta^a &= \frac{1}{2} f_{abc} \theta^b \theta^c \\ s\bar{\omega}^a &= \bar{\chi}^a, & s\bar{\chi}^a &= 0 \end{aligned}$$

$$\begin{aligned}
s\chi^a &= \omega^a, & s\omega^a &= 0 \\
sQ^{ab} &= J^{ab}, & sJ^{ab} &= 0.
\end{aligned} \tag{2}$$

Localize nonlocal terms like  $\varphi^a \frac{1}{\partial^2} \varphi^a$  is a nontrivial task and is done introducing the fields  $\chi, \bar{\chi}, \omega, \bar{\omega}$ , that forms a BRST quartet, i.e  $s\bar{\omega}^a = \bar{\chi}^a$ ,  $s\chi^a = \omega^a$ ,  $s^2 = 0$  and the sources  $Q^{ab}$  and  $J^{ab}$  in a BRST doublet structure[15]. The parameter  $\xi$  has to be introduced since the introduction of the term  $J^{ab}(\bar{\chi}^a - \chi^a)\varphi^b$  gives rise to novel vacuum energy divergences proportional to  $J^2$ .

Here is recommended to spend a few words about the localization of nonlocal operators done with BRST quartets. It is clear that, integrating into the fields  $\bar{\chi}$  and  $\chi$  we have

$$\partial_\mu \bar{\chi}^a \partial^\mu \chi^a + J^{ab}(\bar{\chi}^a - \chi^a)\varphi^b \implies -J^{ab}\varphi^b \frac{1}{\partial^2} J^{ac}\varphi^c. \tag{3}$$

If we find one equation that gave us a nonzero value for the source  $J^{ab}$  the nonlocal operator changes the original Klein-Gordon propagator to a Gribov type one[9]. This task is one of the most important in the Gribov procedure. Unfortunately is not possible to obtain, from geometrical considerations, a gap equation in any case, for example in our present case in which there are no null eigenvalues associated to the equation  $(-\partial^2 + m^2)\varphi = \varrho\varphi$ . In more complex actions like Yang-Mills in the Feynmann gauge, for example, it is also impossible to obtain a gap equation from geometrical considerations and a gap equation is imposed “by hand”. In these sense, the *LCO* procedure together with the localization process appears to be a natural way to obtain nonzero values for  $J$ . For a detailed introduction to the local composite operator (LCO) formalism and to the algebraic renormalization technique, the reader is referred to [17, 18], respectively. Again it is important to emphasize that the scalar model is a toy model and we are using this model to study the procedure of symmetry breaking that can be also applied to the Gribov-Zwanziger action. This procedure is most easily understood with the toy model as a guide, of course not all properties of the Gribov-Zwanziger action are presented in the toy model. For example, in the strong coupling limit of Yang-Mills, the effective coupling constant present in the calculation of the minimum of the potential in Gribov-Zwanziger action is less than one. This is not the case of the scalar field action. We will not deal with the renormalizability of the Gribov-Zwanziger action since it was done into many references, as for example in [9, 10, 16, 19]. However, it is convenient to present the similarities and differences between this action and the toy model presented. For detailed calculations of the renormalizability of the action of Gribov-Zwanziger in the Landau gauge follow [16]. In order to constrain the gauge fields to the Gribov region and have a local action in the fields, a BRST quartet structure  $(\omega_i^a, \varphi_i^a, \bar{\omega}^{ai}, \bar{\varphi}^{ai})$  and a pair of BRST sources are introduced  $(\bar{Q}_\mu^{ai}, Q_{\mu i}^a, \bar{J}_\mu^{ai}, J_{\mu i}^a)$ . The BRST for the fields and sources are:

$$\begin{aligned}
sA_\mu^a &= -(D_\mu c)^a \\
sc^a &= \frac{1}{2} f^{abc} c^b c^c \\
s\bar{c}^a &= ib^a, & sb^a &= 0 \\
s\bar{\omega}^{ai} &= \bar{\varphi}^{ai}, & s\bar{\varphi}^{ai} &= 0 \\
s\varphi_i^a &= \omega_i^a, & s\omega_i^a &= 0 \\
s\bar{Q}_\mu^{ai} &= \bar{J}_\mu^{ai}, & s\bar{J}_\mu^{ai} &= 0 \\
sQ_{\mu i}^a &= J_{\mu i}^a, & sJ_{\mu i}^a &= 0
\end{aligned} \tag{4}$$

These new fields transform under a global  $U(f)$  symmetry on the composite index  $i = (\nu, b)$ , with  $f = 4(N^2 - 1)$ . The localized, BRST invariant version of the Gribov-Zwanziger action is given by:

$$\begin{aligned}
S = \int d^4x_E \{ & \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a - (i\partial_\mu b^a) A_\mu^a - (\partial_\mu \bar{c}^a) (D_\mu c)^a \\
& + (\partial_\mu \bar{\varphi}^{ai}) (D_\mu \varphi_i)^a - (\partial_\mu \bar{\omega}^{ai}) (D_\mu \omega_i)^a + f^{abc} (\partial_\mu \bar{\omega}^{ai}) (D_\mu c)^b \varphi_i^c \\
& + \bar{J}_\mu^{ai} g f_{abc} A_\mu^b \varphi_i^c - \bar{Q}_\mu^{ai} g f_{abc} A_\mu^b \omega_i^c + g f^{abc} \bar{Q}_\mu^{ai} (D_\mu c)^b \varphi_i^c \\
& + J_{\mu i}^a g f_{abc} A_\mu^b \bar{\varphi}^{ci} + Q_{\mu i}^a g f_{abc} A_\mu^b \bar{\omega}^{ci} - g f^{abc} J_{\mu i}^a (D_\mu c)^b \bar{\omega}^{ci} + \xi \bar{J}_\mu^{ai} J_{\mu i}^a - \xi \bar{Q}_\mu^{ai} Q_{\mu i}^a \\
& + \Omega_\mu^a (sA_\mu^a) + L^a(sc^a) \} ,
\end{aligned} \tag{5}$$

where we can see that the sources  $\bar{J}_\mu^{ai}$  and  $J_{\mu i}^a$  are coupled to  $gf^{abc}\bar{\varphi}^{bi}A_\mu^c, gf^{abc}\varphi^{bi}A_\mu^c$ , defining a local composite operator of UV dimension 2, like in the toy model. As a demonstration of the renormalizability of the Gribov-Zwanziger action is extensive and has been held in many gauges, we will pass to the analyses required for the algebraic renormalizability of the toy model.

### 3 Equations compatible with the Quantum Action Principle

Now we will present several symmetries of the action  $\Sigma$  that are compatible with the Quantum Action Principle (QAP)[20], which will be useful in the BRST renormalization procedure. First we have equations of motion with classical breaking.

$$\begin{aligned}\frac{\delta\Sigma}{\delta\bar{\omega}^a} &= \partial^2\omega^a, \quad \frac{\delta\Sigma}{\delta\omega^a} = -\partial^2\bar{\omega}^a + Q^{ab}\varphi^b, \\ \frac{\delta\Sigma}{\delta\bar{\chi}^a} &= -\partial^2\chi^a + J^{ab}\varphi^b - Q^{ab}f_{bcd}\varphi^c\theta^d, \\ \frac{\delta\Sigma}{\delta\chi^a} &= -\partial^2\bar{\chi}^a - J^{ab}\varphi^b + Q^{ab}f_{bcd}\varphi^c\theta^d.\end{aligned}\tag{6}$$

It is important to emphasize that the ghost fields  $\theta^a$  are global and therefore classical fields as well as the sources.

The Slavnov-Taylor Identity:

$$S(\Sigma) = \int d^4x_E \{ f_{abc}\varphi^b\theta^c \frac{\delta\Sigma}{\delta\varphi^a} + \bar{\chi}^a \frac{\delta\Sigma}{\delta\bar{\omega}^a} + \omega^a \frac{\delta\Sigma}{\delta\chi^a} + J^{ab} \frac{\delta\Sigma}{\delta Q^{ab}} \} + \frac{1}{2} f_{abc}\theta^b\theta^c \frac{\delta\Sigma}{\delta\theta^a} = 0, \tag{7}$$

a global ghost equation

$$\begin{aligned}\mathcal{G}^a(\Sigma) &= \Delta^a \\ \mathcal{G}^a &= \frac{\delta}{\delta\theta^a} - \int d^4x_E (f_{abc}Q^{ib} \frac{\delta}{\delta J^{ic}}), \quad \Delta^a = - \int d^4x_E \xi f_{abc}Q^{ib}J^{ic},\end{aligned}\tag{8}$$

the rigid symmetry

$$\mathcal{W}^a(\Sigma) = \int d^4x_E (f_{abc}\{\varphi^b \frac{\delta\Sigma}{\delta\varphi^c} + J^{ic} \frac{\delta\Sigma}{\delta J^{ib}} + Q^{ic} \frac{\delta\Sigma}{\delta Q^{ib}}\}) + f_{abc}\theta^b \frac{\delta\Sigma}{\delta\theta^c} = 0, \tag{9}$$

and a linearly broken symmetry which involves the ghosts of the quartet, the source  $Q^{ab}$  and the global ghost  $\theta^a$

$$\begin{aligned}\mathcal{Q}(\Sigma) &= \Pi \\ \mathcal{Q} &= \int d^4x_E (\bar{\omega}^a \frac{\delta}{\delta\bar{\omega}^a} - \omega^a \frac{\delta}{\delta\omega^a} + Q^{ab} \frac{\delta}{\delta Q^{ab}} + f_{bcd}Q^{ab}\theta^d \frac{\delta}{\delta J^{ab}}) \\ \Pi &= \int d^4x_E \xi f_{bcd}Q^{ab}J^{ac}\theta^d.\end{aligned}\tag{10}$$

These equations, together, provide us all the constraints of the classical action  $\Sigma$  that can be extended to the quantum action.

### 4 Stability of the quantum action

In order to study the stability of the quantum action let us start by presenting the quantum numbers of all fields and sources: We remark to the fact that in the stability analysis of the quantum action it is necessary to take into account that the ghost  $\theta$  is a global ghost and only characterizes the rotation symmetry.

fields and sources	$\varphi$	$\theta$	$\overline{\chi}$	$\chi$	$\overline{\omega}$	$\omega$	$J$	$Q$
UV dimension	1	0	1	1	1	1	2	2
Ghost number	0	1	0	0	-1	1	0	-1
Statistics	co	an	co	co	an	an	co	an

Table 1: Quantum numbers of fields and sources.

#### 4.1 The invariant counterterm

In order to characterize any invariant counterterm which can be added freely to all orders in perturbation theory [20], we perturb the classical action  $\Sigma$  by adding an arbitrary integrated local polynomial  $\Sigma^{count}$  of dimension up-bounded by four, vanishing ghost number and obeying all the other symmetries that are linearly broken. We demand that  $\Gamma = \Sigma + \epsilon \Sigma^{count} + O(\epsilon^2)$ , where  $\epsilon$  is a small expansion parameter, satisfies the same Ward identities as  $\Sigma$ . This requirement provides the following constraints on the counterterm:

$$\frac{\delta \Sigma^{count}}{\delta \overline{\omega}^a} = 0, \quad (11)$$

$$\frac{\delta \Sigma^{count}}{\delta \omega^a} = 0, \quad (12)$$

$$\frac{\delta \Sigma^{count}}{\delta \overline{\chi}^a} = 0, \quad (13)$$

$$\frac{\delta \Sigma^{count}}{\delta \chi^a} = 0, \quad (14)$$

$$\mathcal{B}_\Sigma \Sigma^{count} = 0, \quad (15)$$

$$\mathcal{G}^a(\Sigma^{count}) = 0, \quad (16)$$

$$\mathcal{W}^a(\Sigma^{count}) = 0, \quad (17)$$

$$\mathcal{Q}(\Sigma^{count}) = 0, \quad (18)$$

where in (15),  $\mathcal{B}_\Sigma$  stands for the nilpotent Slavnov-Taylor operator,

$$\mathcal{B}_\Sigma = \int d^4 x_E \{ f_{abc} \varphi^b \theta^c \frac{\delta}{\delta \varphi^a} + \overline{\chi}^a \frac{\delta}{\delta \overline{\omega}^a} + \omega^a \frac{\delta}{\delta \chi^a} + J^{ab} \frac{\delta}{\delta Q^{ab}} \} + \frac{1}{2} f_{abc} \theta^b \theta^c \frac{\delta}{\delta \theta^a} = 0. \quad (19)$$

The set of equations (11,12,13,14) are of particular importance in order to obtain the counterterm action. Due to the fact that they are local equations on the fields of the quartet. Thus the fields  $\overline{\omega}, \omega, \overline{\chi}, \chi$  do not appear in  $\Sigma^{count}$ . Equations (15,16,17,18) are responsible for reducing even more the set of fields and sources from which  $\Sigma^{count}$  may depend. At the end, the counterterm action is only a functional of the field  $\varphi^a$  of the form:

$$\Sigma^{count} = \int d^4 x_E \{ \frac{a_1}{2} \partial_\mu \varphi^a \partial_\mu \varphi^a + \frac{a_2}{2} m^2 \varphi^a \varphi^a + \frac{a_3}{4!} \lambda (\varphi^a \varphi^a)^2 \}. \quad (20)$$

This result is extremely important in the analysis of the quantum behavior of such model. In fact it indicates that all the fields used in the localization process  $\overline{\omega}, \omega, \overline{\chi}, \chi$ , the sources  $J$  and  $Q$  and the parameter  $\xi$  do not renormalizes independently. It is immediate to check that the counterterm action can be reabsorbed into the classical action  $\Sigma$

$$\Sigma + \epsilon \Sigma^{count} = \Sigma(\lambda_0, \xi_0, m_0, \varphi_0^a, \overline{\omega}_0^a, \omega_0^a, \overline{\chi}_0^a, \chi_0^a) + O(\epsilon^2), \quad (21)$$

by redefining mass, sources, couplings and field amplitudes according to

$$\begin{aligned} \lambda_0 &= Z_\lambda \lambda & \xi_0 &= Z_\xi \xi & m_0^2 &= Z_m m^2 \\ \varphi_0^a &= Z_\varphi^{\frac{1}{2}} \varphi^a & & & & \\ \overline{\omega}_0^a &= Z_\omega^{\frac{1}{2}} \overline{\omega}^a & \omega_0^a &= Z_\omega^{\frac{1}{2}} \omega^a & & \\ \overline{\chi}_0^a &= Z_\chi^{\frac{1}{2}} \overline{\chi}^a & \chi_0^a &= Z_\chi^{\frac{1}{2}} \chi^a & & \\ J_0^{ab} &= Z_J J^{ab} & Q_0^{ab} &= Z_Q Q^{ab} & & \end{aligned} \quad (22)$$

with

$$\begin{aligned}
Z_\lambda &= 1 + \varepsilon z_\lambda = 1 + \varepsilon(a_3 - 2a_1) \\
Z_\xi &= 1 + \varepsilon z_\xi = 1 + \varepsilon a_1 \\
Z_m &= 1 + \varepsilon z_m = 1 + \varepsilon(a_2 - a_1) \\
Z_{\frac{1}{\varphi}} &= 1 + \frac{\varepsilon}{2} z_\varphi = 1 + \frac{\varepsilon}{2} a_1 \\
Z_{\frac{1}{\omega}} &= 1 + \frac{\varepsilon}{2} z_\omega = 1 & Z_{\frac{1}{\bar{\omega}}} &= 1 + \frac{\varepsilon}{2} z_{\bar{\omega}} = 1 \\
Z_{\frac{1}{\chi}} &= 1 + \frac{\varepsilon}{2} z_\chi = 1 & Z_{\frac{1}{\bar{\chi}}} &= 1 + \frac{\varepsilon}{2} z_{\bar{\chi}} = 1 \\
Z_J = Z_Q &= 1 + \varepsilon Z_Q = 1 - \frac{\varepsilon}{2} a_1.
\end{aligned} \tag{23}$$

In possession of the renormalization relations, we can now turn to the calculation of the effective potential.

## 5 Effective potential at one loop in $\Sigma$ and similarities with the Gribov-Zwanziger case

The first step in order to study the condensate  $(\bar{\chi}^a - \chi^a)\varphi^b$  is to analyse the generating functional  $W(j)$ . Setting thus to zero the external source  $Q^{ab}$  we have

$$\exp(-W(j)) = \int [D\phi] \exp - \{S_{O(n)} + \int d^4x_E (J^{ab}(\bar{\chi}^a - \chi^a)\varphi^b + \frac{\xi}{2} J^{ab} J^{ab})\}, \tag{24}$$

where  $[D\phi]$  denotes integration over all quantum fields, i.e  $\varphi, \chi, \bar{\chi}, \omega, \bar{\omega}$  and

$$S_{O(n)} = \int d^4x_E \left\{ \frac{1}{2} \partial_\mu \varphi^a \partial_\mu \varphi^a + \frac{m^2}{2} \varphi^a \varphi^a + \frac{\lambda}{4!} (\varphi^a \varphi^a)^2 + \partial_\mu \bar{\chi}^a \partial_\mu \chi^a - \partial_\mu \bar{\omega}^a \partial_\mu \omega^a \right\}. \tag{25}$$

Taking the functional derivative of the above expression we obtain

$$\frac{\delta W(j)}{\delta J^{ab}} \Big|_{J^{ab}=0} = \langle (\bar{\chi}^a - \chi^a) \varphi^b \rangle. \tag{26}$$

Before introducing the Hubbard-Stratonovich field is interesting to note that the following combination of sources and fields has the property of renormalise as exactly the field  $\varphi$

$$\begin{aligned}
\mu^{ab} &= \xi J^{ab} + (\bar{\chi}^a - \chi^a) \varphi^b, \\
\mu_0^{ab} &= Z_\varphi Z_\varphi^{-\frac{1}{2}} \xi J^{ab} + Z_\varphi^{\frac{1}{2}} (\bar{\chi}^a - \chi^a) \varphi^b \\
\mu_0^{ab} &= Z_\varphi^{\frac{1}{2}} \mu^{ab}.
\end{aligned} \tag{27}$$

In order to deal with the term  $\frac{\xi}{2} J^{ab} J^{ab}$  we follow [21], introducing a Hubbard-Stratonovich field  $\sigma^{ab}$  so that

$$\frac{\xi}{2} J^{ab} J^{ab} + J^{ab} (\bar{\chi}^a - \chi^a) \varphi^b = \frac{1}{2\xi} (\mu^{ab} \mu^{ab} - (\bar{\chi}^a - \chi^a) \varphi^b (\bar{\chi}^a - \chi^a) \varphi^b) \tag{28}$$

$$\frac{1}{2\xi} \mu^{ab} \mu^{ab} = -\frac{1}{2\xi} \sigma^{ab} \sigma^{ab} + \frac{1}{\xi} \sigma^{ab} \{ \xi J^{ab} + (\bar{\chi}^a - \chi^a) \varphi^b \}. \tag{29}$$

As a consequence, for the functional generator, we get

$$\exp(-W(j)) = \int [D\phi] \exp - \{S(\varphi, \sigma) + \int d^4x_E \sigma^{ab} J^{ab}\}, \tag{30}$$

with

$$S(\varphi, \sigma) = S_{O(n)} + \int d^4x_E \left\{ -\frac{1}{2\xi} \sigma^{ab} \sigma^{ab} + \frac{1}{\xi} \sigma^{ab} (\bar{\chi}^a - \chi^a) \varphi^b - \frac{1}{2\xi} (\bar{\chi}^a - \chi^a) \varphi^b (\bar{\chi}^a - \chi^a) \varphi^b \right\}, \tag{31}$$

$$\frac{\delta W(j)}{\delta J^{ab}}|_{J^{ab}=0} = \langle (\bar{\chi}^a - \chi^a) \varphi^b \rangle = \langle \sigma^{ab} \rangle. \quad (32)$$

This identity says that the condensate  $\langle (\bar{\chi}^a - \chi^a) \varphi^b \rangle$  is related to the nonvanishing value of  $\sigma^{ab}$  calculated with the action  $S(\varphi, \sigma)$ . It is important to emphasize that due to the intrinsic non perturbative character of the LCO approach it is not necessary to go beyond the one loop level calculations to see the nonperturbative features. Higher loop calculations give us only better numerical results for the value of the condensate, as this is a toy model, numerical refinements are not needed and do not affect the understanding of the proposed method. Again it is important to emphasize here that we know that a scalar model is not asymptotically free. However the Gribov-Zwanziger action has nonabelian gauge fields which have the property of being asymptotically free, as we know from the sign of the beta function. We believe, however, that the toy model can be used to more easily understand the mechanism present in the Gribov-Zwanziger action. Similarly in the Gribov-Zwanziger action we have to do the same kind of analysis in order to calculate the effective potential, *i.e.* setting to zero the sources  $Q_{\mu i}^a$  and  $\bar{Q}_{\mu i}^a$  and repeat the same procedure performed in the toy model.

$$\begin{aligned} \exp(-W_{GZ}(j)) &= \int [D\phi] \exp - \{S_{A, \bar{\varphi}, \varphi, \bar{\omega}, \omega} + S_{\bar{J}, J}\} \\ S_{\bar{J}, J} &= \int d^4 x_E (\bar{J}_{\mu}^{ai} g f_{abc} A_{\mu}^b \varphi^{ci} + J_{\mu i}^a g f_{abc} A_{\mu}^b \bar{\varphi}^{ci} - g f_{abc} J_{\mu i}^a (D_{\mu} c)^b \bar{\omega}^{ci} + \xi \bar{J}_{\mu}^{ai} J_{\mu i}^a) \\ S_{A, \bar{\varphi}, \varphi, \bar{\omega}, \omega} &= \int d^4 x_E \left\{ \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a - (i \partial_{\mu} b^a) A_{\mu}^a - (\partial_{\mu} \bar{c}^a) (D_{\mu} c)^a \right. \\ &\quad \left. + (\partial_{\mu} \bar{\varphi}^{ai}) (D_{\mu} \varphi_i)^a - (\partial_{\mu} \bar{\omega}^{ai}) (D_{\mu} \omega_i)^a + f^{abc} (\partial_{\mu} \bar{\omega}^{ai}) (D_{\mu} c)^b \varphi_i^c \right\}, \end{aligned} \quad (33)$$

The functional derivative with respect to the sources  $\bar{J}_{\mu}^{ai}$  and  $J_{\mu}^{ai}$  gave us

$$\begin{aligned} \frac{\delta W_{GZ}(j)}{\delta J_{\mu}^{ai}}|_{J_{\mu}^{ai}=0} &= g f_{abc} \langle A_{\mu}^b \varphi^{ci} \rangle - g f^{abc} \langle (D_{\mu} c)^b \bar{\omega}^{ci} \rangle. \\ \frac{\delta W_{GZ}(j)}{\delta \bar{J}_{\mu}^{ai}}|_{\bar{J}_{\mu}^{ai}=0} &= g f_{abc} \langle A_{\mu}^b \bar{\varphi}^{ci} \rangle, \end{aligned} \quad (34)$$

and, like in the toy model, we can introduce the adequate Hubbard-Stratonovich fields and rewrite the  $W_{GZ}(\sigma)$  as:

$$\exp(-W_{GZ}(\sigma)) = \int [D\phi] \exp - \{S_{GZ}(\sigma) + \int d^4 x_E (\bar{\sigma}_{\mu}^{ai} J_{\mu}^{ai} + \sigma_{\mu}^{ai} \bar{J}_{\mu}^{ai})\}, \quad (35)$$

with

$$\begin{aligned} S_{GZ}(\sigma) &= S_{A, \bar{\varphi}, \varphi, \bar{\omega}, \omega} \\ &+ \int d^4 x_E \left\{ -\frac{1}{\xi} \bar{\sigma}_{\mu}^{ai} \sigma_{\mu}^{ai} + \frac{1}{\xi} \sigma_{\mu}^{ai} [g f_{abc} (A_{\mu}^b \bar{\varphi}^{ci})] + \frac{1}{\xi} \bar{\sigma}_{\mu}^{ai} [g f_{abc} (A_{\mu}^b \varphi^{ci} - g f^{abc} (D_{\mu} c)^b \bar{\omega}^{ci})] \right\} \\ &- \int d^4 x_E \left\{ \frac{1}{\xi} g^2 f_{abc} A_{\mu}^b \varphi^{ci} f_{ade} A_{\mu}^d \bar{\varphi}^{ei} + \frac{1}{\xi} g^2 f_{abc} A_{\mu}^b \varphi^{ci} f_{ade} (D_{\mu} c)^d \bar{\omega}^{ei} \right\}, \end{aligned} \quad (36)$$

$$\frac{\delta W_{GZ}(j)}{\delta J_{\mu}^{ai}}|_{J_{\mu}^{ai}=0} = \langle \sigma_{\mu}^{ai} \rangle \qquad \frac{\delta W_{GZ}(j)}{\delta \bar{J}_{\mu}^{ai}}|_{\bar{J}_{\mu}^{ai}=0} = \langle \bar{\sigma}_{\mu}^{ai} \rangle. \quad (37)$$

Clearly the difference between the calculation with Gribov-Zwanziger and the toy model is the coupling constant. In the case of Gribov-Zwanziger, the coupling with  $\frac{g^2}{\xi}$  indicates that  $\xi$  is proportional to  $\frac{1}{g^2}$ . Thus, in the infrared limit the effective potential, which is proportional to  $\xi$  allows perturbative calculations and present a well-defined mass gap, indeed the same result is also obtained by the gap equation in Gribov-Zwanziger.

## 5.1 Evaluation of the toy model effective potential at one loop

In order to compute the effective potential for  $\sigma^{ab}$  at the one-loop order only the quadratic part of the action  $S(\varphi, \sigma)$  is relevant, namely

$$S_{quad} = \int d^4x_E \left\{ \frac{\sigma_{ab}\sigma_{ab}}{2\xi} + \phi_a^\dagger \mathcal{M}_{ab} \phi_b \right\} \quad (38)$$

where  $\phi_a^\dagger = (\varphi^a, \bar{\chi}^a)$ , and

$$\mathcal{M}^{ab} = \begin{pmatrix} \frac{1}{2}(k^2 + m^2)\delta_{ab} & -\frac{1}{\xi}\sigma_{ab} \\ \frac{1}{\xi}\sigma_{ab} & k^2\delta_{ab} \end{pmatrix}. \quad (39)$$

After straight forward calculations, using dimensional regularization and in the  $\overline{MS}$  scheme the one loop effective potential is given by:

$$V_{eff} = \frac{\sigma_{ab}\sigma_{ab}}{2\xi} + \hbar^3 \frac{n(n-1)}{128\pi^2} (m^4 - 2 \frac{\sigma_{ab}\sigma_{ab}}{\xi^2(n)}) \left\{ \ln \left( \frac{2 \frac{\sigma_{ab}\sigma_{ab}}{\xi^2(n)}}{16\pi^2 \Lambda^4} \right) - \frac{5}{6} \right\}, \quad (40)$$

whose minimum of the effective potential is given by the condition

$$\sigma_{min}^{ab} \sigma_{min}^{ab} = 4\pi^2 \xi^2 n(n-1) \Lambda^4 \exp\left(\frac{16\pi^2 \xi}{\hbar^3}\right). \quad (41)$$

According now to [21], the parameter  $\xi$  can be computed order by order in the loop expansion  $\xi = \xi_0 + \hbar\xi_1 + \hbar^2\xi_2 + \dots$  and is obtained from the renormalization group equations. This requirement enables the LCO technique to see nonperturbative effects. In fact [21] shows that this coefficient is scheme independent and plays a crucial role in order to obtain a nontrivial vacuum configuration for  $\langle (\bar{\chi}^a - \chi^a)\varphi^b \rangle$ . Let us now proceed with the evaluation of the parameter  $\xi$  at first order in  $\hbar$  requiring that:

$$\Lambda \frac{dV_{eff}(\sigma)}{d\Lambda} = 0 + O(\hbar). \quad (42)$$

Based on the requirement above and following the references[21, 22, 23], after some standard calculations for the method to obtain the LCO parameter  $\xi$

$$\xi_0 = \frac{n(n-1)}{64\pi^2 \gamma_{\varphi^{(1)}}}, \quad (43)$$

$$\gamma_{\varphi^{(1)}} = \frac{(n+2)}{3} \frac{\lambda^2}{12(4\pi)^4} \quad (44)$$

completing therefore the evaluation of the one-loop effective potential for  $(\bar{\chi}^a - \chi^a)\varphi^b$ . Important remarks are now in order to be presented.

- By combining the LCO technique with the BRST algebraic renormalization we have been able to obtain the one-loop effective potential for  $(\chi^a - \bar{\chi}^a)\varphi^b$ . It is clear that in the toy model this potential do not correspond to a stable mass gap due to the differences between the scalar interaction and the Gribov-Zwanziger interaction. However by construction, the effective potential  $V_{eff}(\sigma)$  obeys the renormalization group equation turning possible to determine that the vacuum non zero expectation value is stable in the Gribov-Zwanziger case. It is also clear that, in the Gribov-Zwanziger case the parameter  $\xi_0$  is proportional to the inverse of  $g^2$  i.e.  $\xi_0 \propto \frac{1}{g^2}$ .
- The propagator for  $\varphi$ , in the condensed vacuum, becomes now a Gribov type propagator and is given by:

$$\langle \varphi^a(k) \varphi^b(-k) \rangle = \frac{k^2}{k^4 + m^2 k^2 + \frac{v^4}{\xi^2}}, \quad (45)$$

where

$$\sigma^{ab} = v^2 \delta^{ab}. \quad (46)$$

This result indicates that in an action with BRST quartets and an insertion of  $UV$  dimension 4 a Gribov type propagator will appear, if there is a stable condensed vacuum. We emphasize again that the LCO mechanism allows us to determine when a Gribov type propagator is favored or not simply analyzing the vacuum expectation of the effective potential.



- Due to the characteristics of the *LCO* method, exactly the same results of the usual gap equation can be obtained in the case of the Gribov-Zwanziger action. The method however is applicable to other actions in which the geometrical motivation, given by restriction to the Gribov region, for the gap equation can not be obtained in a direct way. An example of a Yang-Mills action presenting a Gribov type propagator, whose equation of gap should be placed as an extra condition without direct geometric motivation is given by [24].

## 6 How symmetry breaking changes the BRST

Since we have a non zero expectation value for  $\langle (\bar{\chi}^a - \chi^a)\varphi^b \rangle$  and due to equation (26), which relates the value of these condensate to the source  $J^{ab}$ , it is natural to redefine these source as  $J^{ab} = \tilde{J}^{ab} + \frac{v^2}{\xi}\delta^{ab}$ , where  $\tilde{J}^{ab}$  has zero expectation value. Substituting  $\tilde{J}^{ab} + \frac{v^2}{\xi}\delta^{ab}$  in the Slavnov Taylor identity we obtain for the action  $\Sigma_{v \neq 0}$ , which is defined by:

$$\Sigma_{v \neq 0} = \Sigma_{(J + \frac{v^2}{\xi})}|_{J=0, Q=0}, \quad (47)$$

the following Slavnov Taylor relation

$$\begin{aligned} S(\Sigma_{v \neq 0}) &= \int d^4x_E \{ f_{abc} \varphi^b \theta^c \frac{\delta \Sigma_{v \neq 0}}{\delta \varphi^a} + \bar{\chi}^a \frac{\delta \Sigma_{v \neq 0}}{\delta \bar{\omega}^a} + \omega^a \frac{\delta \Sigma_{v \neq 0}}{\delta \chi^a} + (\tilde{J}^{ab} + \frac{v^2}{\xi} \delta^{ab}) \frac{\delta \Sigma_{v \neq 0}}{\delta Q^{ab}} \} \\ &+ \frac{1}{2} f_{abc} \theta^b \theta^c \frac{\delta \Sigma_{v \neq 0}}{\delta \theta^a} \\ &= \int d^4x_E \{ -\frac{v^2}{\xi} \omega^a \varphi^a - \frac{v^2}{\xi} (\bar{\chi}^a - \chi^a) f_{abc} \theta^b \varphi^c \}, \end{aligned} \quad (48)$$

showing that  $\Sigma_{v \neq 0}$  is not invariant under the original BRST symmetry and the breaking term has ultraviolet dimension 2 which characterizes a soft breaking term.

Another equation compatible with the quantum action principle is given by

$$\frac{\delta \Sigma_{v \neq 0}}{\delta \bar{\omega}^a} = \partial^2 \omega^a. \quad (49)$$

This equation can be solved for

$$\omega^a(x) = -(\frac{1}{-\partial^2}) \frac{\delta \Sigma_{v \neq 0}}{\delta \bar{\omega}^a} \equiv - \int d^4y_E \{ (\frac{1}{-\partial^2})_{xy} \frac{\delta \Sigma_{v \neq 0}}{\delta \bar{\omega}^a(y)} \}. \quad (50)$$

The term  $-\frac{v^2}{\xi} (\bar{\chi}^a - \chi^a) f_{abc} \theta^b \varphi^c$  also can be solved with the use of 2 equations compatible with the quantum action principle. The equations are:

$$\begin{aligned} \frac{\delta \Sigma_{v \neq 0}}{\delta \bar{\chi}^a} &= -\partial^2 \chi^a + \frac{v^2}{\xi} \varphi^a \\ \frac{\delta \Sigma_{v \neq 0}}{\delta \chi^a} &= -\partial^2 \bar{\chi}^a - \frac{v^2}{\xi} \varphi^a. \end{aligned} \quad (51)$$

Therefore the breaking term of (48) can be rewritten as

$$S(\Sigma_{v \neq 0}) - \int d^4x_E \frac{v^2}{\xi} \{ (\frac{1}{-\partial^2}) \varphi^a \frac{\delta \Sigma_{v \neq 0}}{\delta \bar{\omega}^a} - f_{abc} \theta^b (\frac{1}{-\partial^2}) \varphi^c (\frac{\delta \Sigma_{v \neq 0}}{\delta \chi^a} - \frac{\delta \Sigma_{v \neq 0}}{\delta \bar{\chi}^a}) \} = 0, \quad (52)$$

and the action  $\Sigma_{v \neq 0}$  is left invariant under the new set of BRST transformations:

$$\begin{aligned} s' \varphi^a &= f_{abc} \varphi^b \theta^c \\ s' \theta^a &= \frac{1}{2} f_{abc} \theta^b \theta^c \\ s' \bar{\omega}^a &= \bar{\chi}^a - \frac{v^2}{\xi} (\frac{1}{-\partial^2}) \varphi^a \end{aligned}$$

$$\begin{aligned}
s' \bar{\chi}^a &= -\frac{v^2}{\xi} f_{abc} \theta^b \left( \frac{1}{-\partial^2} \right) \varphi^c \\
s' \chi^a &= \omega^a + \frac{v^2}{\xi} f_{abc} \theta^b \left( \frac{1}{-\partial^2} \right) \varphi^c \\
s' \omega^a &= 0 \quad (s')^2 = 0.
\end{aligned} \tag{53}$$

The operator  $s'$  exhibits explicit dependence from  $v^2$ . Moreover, it reduces to the operator  $s$  when  $v = 0$ . One should notice that the operator  $s'$  is non-local and representing a symmetry of the action, that is an integrated functional, and cannot be used to analyse the renormalizability properties of the model due to his non-local character. However it can be very useful to calculate the expectation value of exact BRST quantities as presented in [14]. It is also important to realize that  $\bar{\omega}^a \partial^2 \omega^a - \bar{\chi}^a \partial^2 \chi^a$  is no longer cohomologically trivial when we analyze the cohomology of  $s'$ . Thus terms that are trivial by the  $s$  symmetry becomes nontrivial by  $s'$  changing the behavior of the propagator in the infrared limit.

Importantly, a Gribov type propagator has no direct interpretation as particle. Thus the search for composed operators that presents a particle representation, which are the natural candidates to be the physical particles described in this action, are fundamental in order to obtain the physical content of the model. This new set of symmetries despite being nonlocal are nilpotent and can formally define a cohomological problem helping to obtain composite operators that are good candidates to be associated to physical particles. For example with this new symmetry is easy to observe that

$$\bar{\omega}^a \omega^a - \bar{\chi}^a \chi^a - \frac{v^2}{\xi} \frac{1}{\partial^2} \varphi^a \chi^a \tag{54}$$

has zero expectation value and is not a good candidate for a composed operator. In fact is easy to use the property that this term is a BRST variation of

$$s'(\bar{\omega}^a \chi^a) = \bar{\omega}^a \omega^a - \bar{\chi}^a \chi^a - \frac{v^2}{\xi} \frac{1}{\partial^2} \varphi^a \chi^a, \tag{55}$$

thus relating the expected value of  $\frac{v^2}{\xi} \frac{1}{\partial^2} \varphi^a \chi^a$  with  $\bar{\omega}^a \omega^a - \bar{\chi}^a \chi^a$ . This is precisely the property required to obtain a composite operator which may have a particle representation. A good candidate to have a particle representation should not be related to any other composed operator by a BRST transformation. In fact this is a normal property of the Slavnov-Taylor operator i.e variations of the Slavnov-Taylor in relation to fields of the action turn possible to obtain relations among correlators

Let us focus on dimension 2 condensates. According [25] the candidate to be a good composed operator is obtained imposing that the correlator between those operators presents a spectral function that is positive defined. In order to do that the *Källén – Lehmann* representation of the correlator function is calculated for a scalar model. Our model reduces to that presented in the above cited reference if we take the mass  $m = 0$ ,  $\frac{v^2}{\xi} = \mu^2$  and forget all interaction terms. In these case the candidate to be a good composed operator to be associated to a particle representation is given by

$$O = \varphi^a \varphi^a - \frac{1}{\sqrt{2}} (\bar{\chi}^a - \chi^a) (\bar{\chi}^a - \chi^a). \tag{56}$$

This operator is not BRST invariant and his variation is given by:

$$s' O = -(\bar{\chi}^a - \chi^a) \omega^a + 2\mu^2 f_{abc} \theta^b (\bar{\chi}^a - \chi^a) \frac{1}{\partial^2} \varphi^c. \tag{57}$$

This operator is not a BRST variation but we are able to note that

$$\begin{aligned}
s'(O(x)O(y)) &= 0 \\
(O(x)O(y)) &\neq s' \Delta(x, y),
\end{aligned} \tag{58}$$

making the problem of obtaining a particle representation into a cohomology problem. In order to turn more simple to understand this result is useful to remember that the propagators of the toy model are:

$$\begin{aligned}
\langle \varphi^a(k) \varphi^b(-k) \rangle &= \frac{k^2}{k^4 + m^2 k^2 + \mu^4} & \langle \varphi^a(k) \chi^b(-k) \rangle &= \frac{\mu^2}{k^4 + m^2 k^2 + \mu^4}
\end{aligned}$$

$$\begin{aligned}
\langle \varphi^a(k) \bar{\chi}^b(-k) \rangle &= -\frac{\mu^2}{k^4 + m^2 k^2 + \mu^4} & \langle \bar{\chi}^a(k) \chi^b(-k) \rangle &= \frac{k^2 + m^2}{k^4 + m^2 k^2 + \mu^4} \\
\langle \bar{\omega}^a(k) \omega^b(-k) \rangle &= -\frac{1}{k^2}, & \mu^2 &= \frac{v^2}{\xi}.
\end{aligned} \tag{59}$$

The fact that does not exist a mixed propagator  $\langle \varphi^a \omega^b \rangle$ , due to the original quartet structure, turn possible the existence of the above mentioned cohomology problem for the physical correlator. This mechanism of understanding the construction of physical correlators could be a way to see how to generalize the *i - particle* criterium presented in [25] in order to introduce interaction and go beyond the one loop calculation.

## 7 Conclusions

In this work we present a model in which a nonlocal term is introduced in a localized way and the one-loop effective potential for the condensate  $\langle (\bar{\chi}^a - \chi^a) \varphi^b \rangle$  has been obtained by combining the local composite operator (LCO), BRST quartets and an algebraic renormalization. Our results indicate that in a model constructed to obtain a Gribov type propagator, through an insertion of a composite operator, the expectation value of this operator can be obtained directly by the LCO technique and effective potential. This result does not require geometric considerations in order to obtain a gap equation as in the case of Gribov-Zwanziger and can be applied to obtain the expectation value of any condensate of ultraviolet dimension 2 in  $D = 4$  actions. The spontaneous symmetry breaking also opens a window of understanding for the determination of observables whose definition makes sense only in the broken phase. This is achieved through the BRS symmetry in the broken phase, which although it can not be used for determining the renormalizability of the model allows to correlate values expected of operators already known in the unbroken phase with operators that are not null only in the broken phase. This line of work in itself opens the possibility to calculate expected values of several composed operators that correspond to observables in a theory with Gribov-type propagators. We hope to use this mechanism to obtain observables in a gauge model in a future work.

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